

1. INTRODUCTION [pg.3-10] [pg. 1-7 1977]

- **10⁻² sec:** 10¹¹K: electrons, positrons, neutrinos, antineutrinos, photons created out of “pure energy” and annihilated repeatedly, with small number of baryons at ratio of 10⁹ electrons, or positrons, or neutrinos, or photons per baryon (ratio confirmed by CMB data). Density ~4 x 10⁹ gm/cc. [pg.5-6]
- **10⁻¹ sec:** 3 x 10¹⁰K [pg.7]
- **1 sec:** 10¹⁰K [pg.7]
- **14 sec:** 3 x 10⁹K: electrons and positrons annihilations>creations (from photons and neutrinos) – this released energy temporarily slowing the cooling rate. [pg.7]
- **3 minutes:** 10⁹K: density slightly <1gm/cc: 10⁹K: protons and neutrons formed complex nuclei – first deuterium, then helium. At the end of 3 minutes: mostly photons, neutrinos, antineutrinos, and small amount of nuclear material (73% hydrogen, 27% helium) & equally small number of electrons left over from the annihilation. [pg.7-8]
- [Based primarily on measurements of galactic recession and the CMB.] [pg.9]

2. EXPANSION [pg.11-43] [pg. 8-38 1977]

- **1750:** Milky Way believed to be a galaxy [rotating at 250km/sec].
- **1755:** Emmanuel Kant believed that a subset of the nebulae seen were galaxies.
- **1800s (mid):** shifting Fraunhofer lines believed to indicate red shift.
- **1910 (approx.):** Henrietta Swan Leavitt & Harlow Shapley discovered Cepheids’ period variations: absolute luminosities and applied the inverse square law required to yield the apparent luminosities seen thereby measuring the distance to them.
- **1910 – 1920:** Vesto Slipher of the Lowell Observatory found that spectral lines of several nebulae were shifted blue or red – interpreted as Doppler phenomena and indicated Andromeda moving toward Earth at 300km/sec with more distant Virgo cluster receding at ~1000km/sec.
- **1917:** Einstein’s general relativity model of a static homogeneous universe did not show red shifts. He therefore added a Cosmological Constant. Later that year W. de Sitter found another solution that did show red-shifts without the need of a cosmological constant. Einstein remarked that his Cosmological Constant addition was his greatest mistake.
- **1922:** Friedmann found a homogeneous and isotropic solution to Einstein’s original equations forming a workable mathematical basis for modern calculations and indicated ways to calculate open – balanced – or closed universes with the critical density proportional to the square of the Hubble constant. Escape velocity of receding galaxies is another way to determine the correct curves.
- **1923:** Edwin Hubble used Mt. Wilson Observatory to resolve Andromeda - found Cepheid variables in the arms & thereby calculated Andromeda as being 900,000Lys away. Later recalibration yielded its distance as >2x10⁶ Lys - followed Milne’s Cosmological Principle of uniform enlargement between galaxies (appears valid for distances on the order of 100 x 10⁶Lys and for non-relativistic speeds).
- **1929:** Hubble announced discovery - galactic redshifts increase in linear proportion to their distances (based on to their brightest stars). The data was inaccurate, and three years later he calculated “Hubble Constant” at 170Km/sec/10⁶Ly. His work continued but by the end of the 1930s his work was interrupted by WWI (and the limits of Mt. Wilson) – work continued by Allan Sandage at Palomar and Mt. Wilson, ...
- Walter Baade and others recalibrated Cepheid relation leading to H₀ = ~15Km/sec/10⁶Ly.

[Note-1: ~3.3 x 10⁶ LY/Mps; thus 15 x 3.3 = 49.5Km/sec/Mps with the more recent value ~ 70Km/sec/mps.]

[Note-2: ~70Km/sec/Mps (units of 1/sec) inverted -> ~13.968 billion years]

- Redshift and Cepheid data verified by U-235 and U-238 decay calculations. NOW: horizons – the size of the universe is proportional to T^{1/2} – T^{2/3} while distance to the horizon is proportional to T. Thus, for earlier and earlier times, the horizon encloses smaller and smaller portions of the universe. With the universe now at ~10 billion years old, the present horizon is at ~30 billion light years. [pg.33-43]

3. CMB [pg.44-76] [pg. 39-70 1977]

- Penzias and Wilson: $7.35\text{cm noise/signal} = 3.5\text{K}$. Bernard Burk had recently spoken with Ken Turner who told him of a lecture he had attended at Johns Hopkins given by Peebles (following the work of Robert Dicke) arguing that CMB signal should exist at $\sim 10\text{K}$, reasoning that otherwise too much of the primordial hydrogen would have fused into heavier elements. [pg.45-51]
- Late 1940s – a big-bang theory of nucleosynthesis was developed by George Gamow, Ralph Alpher, and Robert Herman that led to a 1948 prediction of a 5K black-body CMB spectrum. [pg.51]

RULES RELATING BLACKBODY WAVELENGTH AND ENERGY

$\lambda=1\text{cm photon} = 0.000124\text{eV}$ (M. Planck) and a photon's λ is inv. prop. to its energy (Einstein)

* Thus, each of the 7.35cm photons detected by Penzias & Wilson = $0.000124\text{eV}/7.35 = 0.000017\text{eV}$.

* Applying Planck's $0.000124\text{eV}/1\text{cm } \lambda$ rule: Sun's surface peak $\lambda=500\text{nm} \rightarrow \text{energy}=0.000124\text{eV}/0.00000050 = 2.48\text{eV}$.

Those 2.48eV photons are sufficiently energetic to drive typical 1-2eV chemical reactions, including those of photosynthesis and the photochemical reactions within our visual receptors.

RULES RELATING BLACKBODY TEMPERATURE, WAVELENGTH, AND ENERGY

A photon's energy is proportional to the temperature.

A photon's wavelength is inversely proportional to the temperature.

A blackbody's energy density is proportional to the $(\text{temperature})^4$.

A blackbody's peak energy density $\lambda = 0.29\text{cm/K}$. (Wein's Displacement Law)

For room temperature photons of $\sim 300\text{K} \rightarrow \lambda = .29\text{cm}/300 = 0.00097\text{cm} = 9700\text{nm (infrared)}$,

For solar surface photons of $\sim 5800\text{K} \rightarrow \lambda = 0.29\text{cm}/5800 = 0.000050\text{cm} = 500\text{nm (visible)}$

Boltzmann's Constant: $1.380649 \times 10^{-16}\text{ergs/K} = 8.617333262145 \times 10^{-5}\text{eV/K} = 1.380649 \times 10^{-23}\text{J/K}$

Energy Conversions $1\text{eV} = 1.602 \times 10^{-12} \text{ ergs} = 1.602 \times 10^{-19} \text{ Joules}$

Proof: $1\text{J} = 1\text{Coulomb of charge moved thru a 1V potential, \& 1eV is the work of moving 1 electron thru a 1V potential.}$

As a single electron carries a charge of 1.60×10^{-19} Coulombs, 1eV therefore = 1.60×10^{-19} Joules

CMB-LSS PHOTONS

$$\begin{aligned} E = hf = hc/\lambda &\quad \text{where } h=6.63 \times 10^{-34} \text{ Joule*secs, } c=3 \times 10^8 \text{ m/sec, and } 1\text{eV} = 1.602 \times 10^{-19} \text{ Joules} \\ &= [(6.63 \times 10^{-34} \text{ J*sec}) \times (3 \times 10^8 \text{ m/sec})] / (91 \times 10^{-9} \text{ m}) = 2.185714 \times 10^{-18} \text{ J} \\ &= 2.185714 \times 10^{-18} \text{ J} / 1.602 \times 10^{-19} \text{ J} * \text{eV}^{-1} \rightarrow \text{13.6eV} \end{aligned}$$

CMB-LSS photons decoupled once they fell below the energy required to ionize atomic hydrogen ($\lambda=91\text{nm}$) = **13.66eV**, thereby breaking free of the thermal equilibrium and filling space with the 3000K black body spectral image at ave. $\lambda = \sim 1$ micron and average distance between photons also = ~ 1 micron. With space now having expanded by $\sim 1000x$, the ave. λ has lengthened by $\sim 1000x$, now received at ~ 1 mm with an equivalent T drop of $1000x$ from 3000 to 3K .

The peak CMB-LSS photons now received at wavelength = 1.065mm have been redshifted by $z=1090$, having departed the LSS at $1.065 \times 10^{-3}\text{m} / 1.090 \times 10^3 = 0.9752 \times 10^{-6}\text{m} = 975.2\text{nm}$, which is over 10x longer than the **91nm photons required to ionize hydrogen**. The 3000K spectrum peaking at 975.2nm still had a short-wavelength tail with a diminishing number of the hydrogen ionizing 91nm (UV) photons.

PHOTON ENERGIES AND REACTIONS (CHEMICAL / IONIZATION / NUCLEAR)

Chemical reactions are driven at 1-2eV; Hydrogen atoms are ionized at 13.6eV; Nuclear reactions occur at $\sim 10^6\text{eV}$. (...explaining why a pound of fissionable material is equivalent to about 10^6 pounds of chemical explosive.)

BLACK BODY ENERGY DENSITY CALCULATIONS

Black body peak wavelength is inv. prop. to the temp; thus, the ave. distance between photons is inv. prop. to temp.

The # of things in a fixed volume is inv. prop. to the (ave. separation) 3 , thus for black-body radiation, the # photons in a given volume is proportional to (temp) 3

The energy density (energy/liter) is (photons/liter) x (ave. photon energy). As # photons/liter is prop. to (temp) 3 and the ave. photon energy is prop. to the temp, it follows that **black-body energy/liter is prop. to (temp.) 3 x temp = (temp.) 4**

Stefan-Boltzmann: $4\sigma T^4/c \rightarrow 1\text{K black-body yields} = 4.72\text{eV/liter}$. The energy density then rises as T^4 .

I.e., given that the energy density of a black-body spectrum is 4.72eV/L at a temperature of 1K , then at 10K , the

energy density rises to $4.72\text{eV/L} \times 10^4 = 47,200\text{eV/L}$, and so on. The $\sim 3\text{K}$ noise detected by Penzias & Wilson had energy density = $4.72\text{eV/L} \times 3^4 = 380\text{eV/L}$. When the temp. was 10^3 x larger (3000K), energy density was 10^{12} x greater.

- With Penzias and Wilson's detection at 3.5K, it became important to verify whether or not this represented a component of a black-body spectrum as predicted. Shortly after the original finding, Roll and Wilkinson found a radiation background signal at 3.2cm = 2.5-3.5K. The intensity at 3.2cm was greater than at 7.35cm by the exact amount predicted by the Planck formula. Subsequent experiments have measured a band of wavelengths from 73.5cm – 0.33cm – all consistent with the Planck Black-body formula of a peak between 2.7K – 3K. The Planck formula indicates the distribution maximum should lie at 0.29cm/K, which for 3K works out to just under 0.1cm... thus all measurements by 1965 were above this max. and had not yet fully verified the spectral shape. The finding that a drop-off occurs for longer wavelengths is also consistent with the Rayleigh-Jeans region whereby it is more "difficult" to place longer wavelengths in a fixed space than shorter wavelengths... thus the black-body spectral signature was suggested, but not yet proven. [pg.66-67]

What is the appropriate mathematics to determine a blackbody temperature of 3.5K from Penzias and Wilson's finding excess noise at 4080MHz = wavelength of 7.35cm. (Planck formula yields a maximum wavelength at 3K of $\sim 0.1\text{cm}$, far from the 7.35cm received at 4080MHz.)

- To detect wavelengths of $<0.3\text{cm}$, which is toward the infrared spectrum, one must observe from above the atmosphere. [pg.67]
 - 1941 W.S. Adams and A. McKellar, in researching a gas cloud rich in the CN radical within the constellation Ophiuchus, backlit by the star ζ Oph (Zeta Ophiuchi). The cloud absorbs photons in specific wavelengths that indicate the CN radicals were excited to generate a rotational state that was consistent with being bathed in a perturbation of equivalent temperature of $\sim 2.3\text{K}$. Following the apparent black-body signature in 1965, it was recognized that that this 2.3K signature = 0.263cm wavelength fit the model and was of a shorter wavelength than terrestrial observations had reached, although still not down to the $<0.1\text{cm}$ wavelengths needed to verify the expected black-body signature where a rapid fall off should be seen.
 - In 1974 observations of another CN radical rotational state generated by a wavelength of 0.132cm was detected and appeared to follow the beginning of a drop off, consistent with a $\sim 3\text{K}$ max black-body spectrum. Other early 1970s experiments that probed the $<0.3\text{cm}$ infrared region includes: Cornell U (first finding non-consistent results which were then upgraded), an M.I.T. balloon group, and in 1976 a UCB balloon group – all found a fall off of energy density for short wavelengths in the range of 0.26cm – 0.06cm, consistent for a black-body spectrum max. at $\sim 3\text{K}$. [pg.68-69]
- At the time of the writing of this book – there had been no space-based research undertaken although it was proposed that there should exist small variations in the otherwise isotropic black-body spectrum that lead to the galactic structure in the present universe. Also, it was noted that as the earth rotates around the sun at 30km/sec and the galaxy rotates at $\sim 250\text{km/sec}$ – that signature should be detectable as well. [pg.72]
- The 3K signature allows calculation of a crucial number: the number of photons per unit volume is inversely proportional to the cube of a typical wavelength and hence directly proportional to the cube of the temperature. At 1K there would be 20,282.9 photons/liter, so... at a 3K background there should be 550,000 photons per liter. The density of nuclear particles in the present universe is between 6 – 0.03 particles per 1000 liters (with the upper limit being twice the critical density for the universe). Thus, there are between 1×10^8 - 20×10^9 photons for every nuclear particle (neutrons and protons) in the universe today, and this ratio would have been constant from the time when individual photons were being created and destroyed to the present time. (For ease of discussion, this ratio is considered to be **$\sim 10^9$ photons per nuclear particle.**) [pg.73]

Wouldn't this ratio change over time as larger nuclei are created by stellar processes?

- "In order for gravitation to produce the clumping of matter into isolated fragments that had been envisioned by Newton, it is necessary for gravitation to overcome the pressure of matter and the associated radiation. The gravitational force within any nascent clump increases with the size of the clump, while the pressure does not depend on the size; hence at any given density and pressure, there is a minimum mass

which is susceptible to gravitational clumping. This is known as the ‘Jeans mass,’ ” because it was first introduced in theories of the formation of stars by Sir James Jeans in 1902.” [pg.74]

- Jeans mass is proportional to Pressure^{3/2} (see note 5, pg. 176). At 3000K the radiation pressure was enormous and the Jeans mass was correspondingly high (about 10⁶ times larger than the mass of a large galaxy), thus galaxies or even clusters of galaxies could not have been massive enough to have formed at that early time. [pg.74-75]
- Once electrons joined with nuclear particles to form atoms, photon pressure became unlinked. At a given temperature and density, the pressure of matter and radiation is simply proportional to the number of particles or photons, respectively, so when radiation pressure became ineffective, the total effective pressure dropped by a factor of about 10⁹, and the Jeans mass dropped by 3/2 power of that factor to about 10⁻⁶ the mass of a typical galaxy. From that point on, the pressure of matter alone was insufficient to resist matter clumping into galaxies. [pg.75]

What pressure is this referring to, and what is its mechanism? How is Jeans mass calculated?

Using Jeans mass calculations and the isotropy of the universe at the CMB-LSS, have calculations indicated when galaxies should have first formed? Is this consistent with observation?

- The current universe is dominated by mass energy over photon energy: the mass of a nuclear particle is $\sim 939 \times 10^6$ eV (by $E=mc^2$) whereas the average energy of a photon within a 3K black-body radiation spectrum is ~ 0.0007 eV ... thus despite a ratio of 10⁹ photons per matter particle, the energy ratio of mass: photons is on the order of $(1 \times \sim 939 \times 10^6) : (0.0007 \times 10^9)$ or about 10⁹ energy as mass to 0.7 x 10⁶ energy as photons (or $\sim 1400:1$). [pg.75]
- At earlier times, the temperature was higher so the energy of each photon was higher while the energies of neutrons and protons have remained constant. For radiation energy to dominate mass energy, the mean energy of a black-body photon must exceed a nucleon’s energy by about 1 part in 10⁹, or ... about 1eV, which was the case when the temperature was $\sim 1,300$ times the present temperature, or at ~ 4000 K. This marks the time when the universe’s energy content changed from being radiation dominated to the current state of mass dominance. [pg.76]
- “It is striking that the transition from a radiation – to a matter-dominated universe occurred at just about the same time that the contents of the universe were becoming transparent to radiation, at about 3,000K. No one really knows why this should be so...” [pg.76]

Is there any new thinking about the seemingly unconnected facts that 1) the universe transitioned from being radiation dominated to being matter dominated at 4000K at \sim the same time as 2) CMB photons escaped the thermal equilibrium in effect above 3000K? i.e., these seem disconnected: 1) nuclear energy (mass) is fixed in density, while radiation waves “stretch” as space expands; and 2) EM forces interact such that 13.6eV is the minimum energy required to ionize hydrogen atoms. Is there any known reason why these should be related to a very narrow temperature range of between 3000K – 4000K?

- If there were now 10¹⁰ photons per nucleon, then radiation would have continued to predominate over matter until the temperature dropped to 400K, well after the contents of the universe became transparent. [pg.76]

4. RECIPE FOR A HOT UNIVERSE [pg.77-100] [pg. 71-93 1977]

- From after the first few minutes (when the temperature fell to a few billion K) until about the time the universe became transparent to black-body photons, it was highly – essentially entirely – dominated by radiation. (Before the first 3mins, during the quark-gluon period, matter pressure could not be ignored.) [pg.78]
- The average wavelength of a black-body spectrum is inversely proportional to its temperature: the temperature would have decreased in inverse proportion to the size of the universe. [pg.79]
- “While a photon was in free flight between collisions, its wavelength would have increased in proportion to the size of the universe, and there were so many photons per particle that the collisions simply forced the

matter temperature to follow the radiation temperature, not vice versa. Thus, for instance, when the universe was ten thousand times smaller than now, the temperature would have been proportionally higher than now, or about 30,000° K. So much for the true era of radiation.” [\[pg.79\]](#)

[What does Weinberg mean by this last sentence “So much for the true era of radiation?](#)

- The average wavelength of a black-body spectrum is inversely proportional to its temperature: the temperature would have decreased in inverse proportion to the size of the universe. [\[pg.79\]](#)
- At thermal equilibrium, the numbers of matter particles will equal the number of photons and were equally important as radiation in determining the rates of various nuclear reactions at that time, and in determining the rate of expansion of the universe. (Reasoning: “If there are fewer particles than photons, they will be created faster than they are destroyed, and their number will rise; if there are more particles than photons, they will be destroyed faster than they are created and their number will drop.”) [\[pg.79, 84\]](#)
- At some temperature, two photons of sufficient energy could collide and create 2 matter particles by $E=mc^2$. **Boltzmann’s constant = 0.00008617 eV/K**, can be used to determine the needed temp. at which photons could collide and create matter particles. (E.g., at 3000K the characteristic energy of each photon = 0.26eV.) The temp. to create a particle of mass = **m is given by $mc^2/Boltzmann's\ const.$** (E.g., a set of photons can create an e^-/e^+ pair when each of the photons are of an energy set by a minimum temp. $T_{min} = (0.511003 \times 10^6 \text{ eV}) / (0.00008617 \text{ eV/K}) = 6 \times 10^9 \text{ K}$. (An enormously high temperature as evidenced by calculating that the Sun’s core has a temp. of approx. $15 \times 10^6 \text{ K}$. [\[pg.82\]](#)

Rest Mass Entity	Wavelength	Energy in eV	Threshold temp in K
Electrons (e)		0.511003×10^6	6×10^9
Muons (μ)		105.6596×10^6	1.2×10^{12}
Protons (p)		938.26×10^6	1.0888×10^{13}
Neutrons (n)		939.55×10^6	1.0903×10^{13}

Radiation Entity	Wavelength	Energy in eV	Threshold temp in K
HF	>10	< 0.0001	< 0.03
Microwaves	0.01 – 10	0.0001 - <0.03	0.03 - 30
Infrared	0.0001 – 0.01	0.01 – 1	30 – 3,000
Visible	$2 \times 10^{-5} – 10^{-4}$	1-6	3,000 - 15,000 (sun peak 5800K)
UV	$10^{-7} – 2 \times 10^{-5}$	6-1000eV	15,000 – 3,000,000
XRay	$10^{-9} – 10^{-7}$	1,000-100,000	$10^6 – 10^8$
GammaRay	<10 ⁻⁹	>100,000	>10 ⁸

- At temps. > threshold, a particle’s average energy is $\sim =$ to the temp*Boltzmann’s constant. [\[pg.84\]](#) and at high above their threshold temps., material particles behave as photons, with their average energies \gg than in their rest mass energy alone; their rest mass can therefore be ignored in calculations - leaving their energy densities as proportional to the T^4 (as is true for photons). At those early times, the energy density of the universe at any time is proportional to T^4 , and to the number of species of particles whose threshold temperature is below the current temperature. In total energy density calculations, particles and antiparticles are counted as separate species. Similarly, photons and fermions that have 2-types of spins are counted as separate species. Electrons, neutrinos, muons, Protons, and Neutrons all obey the Pauli Exclusion Principle, which effectively lowers their contribution to the total energy by a factor of 7/8ths. [\[pg.85\]](#)

Entities	Types	Numbers of Species		Mean half-lives
		Types x Spins x Pauli Exclus. Total Entropic Count		
Photons	: γ	$1 \times 2 \times 1$	= 2	stable
Neutrinos	: ν_e, ν_μ, ν_τ	$2 \times 1 \times 7/8$	= $7/4$	stable
	: ν_{μ}, ν_τ	$2 \times 1 \times 7/8$	= $7/4$	stable
	: ν_e, ν_τ	$2 \times 1 \times 7/8$	= $7/4$	stable \leftarrow Not in book/discovered in 2000
Electrons	: e^-, e^+	$2 \times 2 \times 7/8$	= $7/2$	stable
Muons	: μ^-, μ^+	$2 \times 2 \times 7/8$	= $7/2$	$2.197 \times 10^{-6} \text{ s}$
Pi ₀ mesons	: π^0	$1 \times 1 \times 1$	= 1	$0.8 \times 10^{-16} \text{ s}$
Pi _{+/} mesons	: π^+, π^-	$2 \times 1 \times 1$	= 2	$2.6 \times 10^{-8} \text{ s}$
Protons	: p, p^-	$2 \times 2 \times 7/8$	= $7/2$	stable
Neutrons	: n, n^-	$2 \times 2 \times 7/8$	= $7/2$	920s

How does the Pauli Exclusion Principle result in electrons, muons, (tau), neutrinos, Protons, and Neutrons to add only 7/8'ths to the total energy density than if the rule didn't apply?

(I thought that the Pauli Exclusion Principle relates to standing waves within atomic species only.)

Given we now know there are 3 types of neutrinos (electron, muon, tau), should the table and subsequent calculations be updated to reflect their total entropic species = additional 7/4?

- It is the balance between the gravitational field and the outward momentum that governs the rate of expansion of the universe. And, it is the total energy density that provides the gravitational field, which depends essentially only on temperature. So, the temperature can be used as a clock – cooling over time as the universe expands. Therefore, **if no thresholds are crossed, the time it takes for the universe to fall from one density value to another is proportional to the difference of the inverse square-roots of the two energy densities under evaluation.** (see note 3, ppg. 170-172). But, the energy density is proportional to $(\text{temperature})^4$ and to the number of particle species with threshold temperatures below the actual temperature. Hence, **it is also true that “as long as the temperature does not cross any threshold values, the time it takes for the universe to cool from one temperature to another is proportional to the difference of the inverse squares of these temperatures.** [pg.86]

Example given: Start at $T = 10^8\text{K}$ finding it takes 0.06 years from $10^8 \rightarrow 10^7\text{K}$ (a factor of 10^{-1} difference).
It will take $1/(10^{-1})^2 = 100$ x that time for the next 10^{-1} step from $10^7 \rightarrow 10^6 = 6$ years.
It will take $1/(10^{-1})^2 = 100$ x that time for the next 10^{-1} step from $10^6 \rightarrow 10^5 = 600$ years.
... carrying out the calculations further and adding them up: from $10^8\text{K} \rightarrow 3000\text{K}$ took $\sim 700,000$ y.

How was the starting point of the above calculation obtained, which begins at 108K running to 107K said to have taken 0.06y (22 days)? At some point there must be checkpoint... if that is the CMB-LSS at 3000K, said to have taken $\sim 700,000$ years, but is now believed to have taken 384,000 years, there is an error factor of $700,000/384,000 = 1.82x$ in this approximation.

- Note, when the temperature fell to below 10^9K , all particles and antiparticles might have been expected to annihilate... they clearly did not. [pg.87]
- “The really remarkable thing about a system in thermal equilibrium is that all its properties are uniquely determined once we specify the values of the conserved quantities.” [pg.89]

Discussing conserved quantities, water used as an example, which partially dissociates into H^+ and OH^- . Stated: “there is one hydrogen ion for about every five hundred million water molecules [at STP]. Calculation: 1 mole $\text{H}_2\text{O} = 18\text{gr}$ and 1 liter of $\text{H}_2\text{O} = 1000\text{gr}$ therefore: $1000\text{g} * \text{liter}^{-1} / 18\text{g} * \text{mole}^{-1} = 55.5 \text{ moles/liter}$. Water dissociates such that hydrogen ion concentration = 10^{-7} ($\text{pH} = -\log[10^{-7}] = 7$). Thus, in a water solution there are 55.5 moles of water per liter and 10^{-7} moles of H^+ per liter: $55.5\text{moles H}_2\text{O}/10^{-7}\text{moles H}^+ = 55.5 \times 10^7 \text{ H}_2\text{O to 1 H}^+ = 550 \text{ million to 1 ratio}$. Weinberg notes that this ratio will shift with changing pressures/changing densities – meaning these need to be specified in calculating conserved quantities. [pg.90]

- There appear to be only three conserved quantities whose densities must be specified in a recipe for the early universe: 1) Electric Charge; 2) Baryon Number; 3) Lepton Number. These quantities per unit volume vary inversely with the (size of the universe)³ just as do the numbers of photons per unit volume. And, the photon numbers per unit volume is proportional to the $(\text{temperature})^3$. And, the temperature varies with the inverse size of the universe. **Therefore, the charge, baryon number and lepton number per photon remains fixed.** [pg.93-94]
- Thus, the recipe for the universe can be given by specifying the conserved quantity values as a ratio of them to the number of photons. Note – “... the quantity that varies as the inverse cube of the size of the universe is not the number of photons per unit volume, but the *entropy* per unit volume.” But, as the number of particles is very close to the number of photons, they can be used instead of entropy calculations to good approximation. [pg. 94]
- Rather than considering charge, baryon#, and lepton number per photon, it is more accurate to consider these conserved quantities as ratios to entropy value rather than as ratios to photons. “However, even at very high

temperatures the number of material particles is at most of the same order of magnitude as the number of photons, so we will not be making a serious error if we use the number of photons instead of the entropy as our standard of comparison." The question then is how is entropy calculated?

- Note: if the earth and sun had an excess of positive over negative by 1 part in 10^{36} , the resulting repulsion would overcome gravity – this is strong argument as to the cosmic charge per photon being zero. i.e., the net charge of the universe must equal zero, otherwise [**assuming the universe is finite**] the lines of electrical force would wrap round and round the universe building up an infinite field. [pg.94-95]
- The baryon number in the present universe is equal to the number of nuclear particles: 1 per 10^9 photons. [pg. 95]

Is it possible that there are domains within the universe where antimatter exceeds matter by 1 part in 10^9 – balancing our region of matter over antimatter? Are there any spectral, or other types, of signals that could possibly indicate that to us?

Weinberg stated that it is possible that production of new photons by some unknown frictional or viscosity effect may theoretically have occurred at some time and place that would have shifted the conserved numbers. Has anyone seriously considered this possibility? [pg.96-97]

- Standard Model assumption ("one of the least certain of the assumptions that go into the "standard model": lepton number (particles – antiparticles) per photon is very small. [pg.99]

Thus, the recipe for the contents of the early universe:

"Take a charge per photon equal to zero, a baryon number per photon equal to one part in 1,000 million, and a lepton number per photon uncertain but small. Take the temperature at any given time to be greater than the temperature 3°K of the present radiation background by the ratio of the present age of the universe to the size at that time. Stir well, so that the detailed distributions of particles of various types are determined by the requirements of thermal equilibrium. Place in an expanding universe, with a rate of expansion governed by the gravitational field produced by this medium. After a long enough wait, this concoction should turn into our present universe."

5. THE FIRST THREE MINUTES [pg.101-121] [pg. 94-113 1977]

TIME = 10^{-2} seconds

CHARACTERISTIC EXPANSION TIME = 0.02 seconds

TEMP = 10^{11} Kelvin

CIRCUMFRENCE = prop. to T -> = ($\sim 92 \times 10^9$ LY) $\times (2.72\text{K}/10^{11}\text{K}) = 2.5$ LY

- Everything in the universe is travelling just at the escape velocity from an arbitrary center.
- Before this time, temperatures would have exceeded $1.5 \times 10^{12}\text{K}$ and would have allowed large numbers of pi-mesons to be created and their self-interactions would make calculations difficult. (The exchange of pi-mesons is responsible for most of the attractive force that bind nucleons together.) At this time and temperature, Protons, Neutrons, and muons could no longer be created from photon interactions. [pg.101-102]
- Despite the rapid expansion occurring, particles and radiation are in nearly perfect thermal equilibrium.
- Conserved quantities: charge, baryon number, lepton number are very small or zero. [pg.102]
- Abundant particles all in thermal equilibrium include: electrons, positrons with threshold temperatures well below 10^{11} along with massless particles: photons, neutrinos, and anti-neutrinos. Given electrons and positrons are far above their threshold temperatures, they behave like radiation. [pg.103]
- The total energy density components consist of: electrons=7/8, positrons=7/8, neutrinos=7/8, anti-neutrinos=7/8, and photons=1), summed = 9/2 times the energy density of the photons alone. [pg.103]

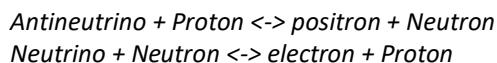
- Applying the Stefan-Boltzmann law (*energy density of black-body radiation = 4.72eV/liter at 1K and is directly prop. to T^4*) -> energy density for EM radiation at $10^{11}\text{K} = 4.72\text{eVx}(10^{11})^4 = 4.72 \times 10^{44}\text{eV/liter}$, so the total energy density at that temp. = $9/2$ times that value = **$21.24 \times 10^{44}\text{eV/liter}$** . This is equivalent (by $1\text{eV}/c^2 = 1.782661921 \times 10^{-36}\text{kg}$) to a mass density of $(21.24 \times 10^{44}\text{eV/liter}) \times (1.782661921 \times 10^{-36}) = \sim 3.8 \times 10^9\text{kg/liter}$, which equals $\sim 3.8 \times 10^9$ times the density of water (at STP). [pg.103-104]
- Everything is traveling at \sim escape velocity from an arbitrary center with the **characteristic time for expansion** defined as “100 times the length of time in which the size of the universe would increase by 1%.” “...characteristic expansion time at any epoch is the reciprocal of the Hubble ‘constant’ at that Epoch.” [pg.104]

See page 157-160 in the 1977 edition and page 170-173 in 1993 edition. This is not clear to me. Also, these books reference H_0 as being $15\text{km/sec}/10^6\text{LY}$ ($x 3.26 \times 10^6\text{LY/mps} = 48.9\text{km/sec/mps}$) which is about 69% of the current rough value of 70km/sec/mps . This then changes the characteristic time for expansion values listed.

- “... the age of the universe is always less than the characteristic expansion time, because gravitation is continually slowing down the expansion.” [pg.104]

This was written in 1977-78, 1993 ... pre-dark-energy discovery... how is this to be interpreted?

- 1 nucleon exists per 10^9 photons or leptons. [pg.104]
- Neutrons are $1.293 \times 10^6\text{eV}$ heavier than Protons. The characteristic energy of leptons is given by (Boltzmann’s constant) * (T); at 10^{11}K they each have energies of $\sim 10^7\text{eV}$, which – combined with their numbers - are much higher than the nucleons and so cause rapid transitions of Protons to Neutrons and Neutrons to Protons. The most important reactions are:



These reactions are at thermal equilibrium and therefore exist at about equal numbers [pg.105]

- Nuclei cannot yet form given they are immediately broken up at temperatures of $6-8 \times 10^6\text{eV}$. [pg.105]
- If H_0 is taken as $15\text{km/sec}/10^6\text{LYs}$ and the universe is taken to be closed, then its circumference currently = $125 \times 10^9\text{LYs}$ and, as the temperature of the universe falls in inverse proportion to its size, the circumference of the universe was less than that by ratio of $T=10^{11}\text{K}$ at 0.02 seconds to the present $T=3\text{K}$, yielding the universe’s circumference at 0.02 seconds = $\sim 4\text{LYs}$. [and radius = 0.637LYs] [pg.105-106]

This was written in 1977-78, 1993 ... Weinberg wrote: “This estimate is based on the present value of the Hubble constant, under the supposition that the density of the universe is about twice its ‘critical density’ value.” [pg.105-106]
Approximately when did the consensus change that Ω appears to be = 1?

TIME = 0.11 seconds

CHARACTERISTIC EXPANSION TIME = 0.20 seconds

TEMP = 3×10^{10} Kelvin

CIRCUMFERENCE = prop. to T -> = $(\sim 92 \times 10^9 \text{ LY}) \times (2.72\text{K} / 3 \times 10^{10}\text{K}) = 8.3 \text{ LY}$

- Contents remain dominated by electrons, positrons, neutrinos, antineutrinos, and photons – all in thermal equilibrium and all high above their threshold temperatures – all behaving as radiation therefore energy density dropped as T^4 . To about 30×10^6 times the energy density contained in the rest mass of water. [pg.106]
- Rate of expansion has dropped by $T^{1/2}$ so that the characteristic expansion time has lengthened to about 0.2 seconds. [pg.106]
- Nuclei cannot yet form, but now more heavy neutrons turn into lighter protons than the reverse causing the nucleon balance to shift to 38% neutrons and 62% protons. [pg.106]

TIME = 1.09 seconds

CHARACTERISTIC EXPANSION TIME = ~ 2 seconds

TEMP = 10^{10} Kelvin

CIRCUMFRENCE = prop. to T -> = $(\sim 92 \times 10^9 \text{ LY}) \times (2.72 \text{ K} / 10^{10} \text{ K}) = 25 \text{ LY}$

- The decreasing density and temperature have increased the mean free path of neutrinos and antineutrinos to such a degree that they are beginning to behave as free particles, no longer in full thermal equilibrium with electrons, positrons, and photons. They are therefore no longer involved in the transformation of other particles, but remain gravitational components. Before the decoupling, typical neutrino wavelengths were inversely proportional to the temperature and since the temperature was falling off in inverse proportion to the size of the universe, the neutrino wavelengths were increasing in direct proportion to the size of the universe. [\[pg.106-107\]](#)
- Total energy density continued to fall by T^4 , now to an equivalent mass density of 380,000 times that of water. [\[pg.107\]](#)
- Temperature is now only twice the threshold temperature of electrons and positrons, so they are just beginning to annihilate more rapidly than they can be created out of radiation. [\[pg.107\]](#)
- The temperature is still too high for nuclei to form, but the decrease in temperature has now shifted the proton/neutron balance to shift to 24% neutrons and 76% protons. [\[pg.107\]](#)

TIME = 13.82 seconds

CHARACTERISTIC EXPANSION TIME = ????? seconds

TEMP = 3×10^9 Kelvin

CIRCUMFRENCE = prop. To T -> = $(\sim 92 \times 10^9 \text{ LY}) \times (2.72 \text{ K} / 3 \times 10^9 \text{ K}) = 83.4 \text{ LY}$ (pg.106 used different $1/H_0$)

- Below the electron and positron threshold temperature therefore rapidly self-annihilating and the energy released in that process slowed the universe's rate of cooling. As neutrinos are no longer in thermal equilibrium, they do not receive that heat and are presently 8% cooler than electrons, positrons, and photons. [\[pg.108\]](#)
- The energy density is now less than it would be if it had continued to fall by the T^4 rule. [\[pg.108\]](#)
- Now cool enough for stable nuclei like He^4 to gradually form, but this doesn't immediately occur. Because of the still rapid expansion, nuclei can only form in a series of fast two-particle reactions: [\[pg.108\]](#)



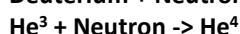
[Deuterium is only loosely bound / rate limiting]



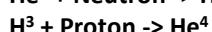
[Helium-3 is not tightly bound]



[Tritium is not tightly bound]



[Helium-4 is stable at these temperatures]



- Neutron \rightarrow Proton conversion slowing but continues w/ balance now 17% Neutrons, 83% Protons. [\[pg.109\]](#)

TIME = 3 minutes 2 seconds

CHARACTERISTIC EXPANSION TIME = ????? seconds

TEMP = 10^9 Kelvin

CIRCUMFRENCE = prop. To T -> = $(\sim 92 \times 10^9 \text{ LY}) \times (2.72 \text{ K} / 10^9 \text{ K}) = 250 \text{ LY}$

- Electrons & positrons have mostly disappeared leaving mostly photons, neutrinos & antineutrinos. [\[pg.109\]](#)
- Photon temps. are now 35% higher than the neutrinos as they remain out of thermal equilibrium. [\[pg.109\]](#)
- Lower temperatures allow both He^3 and He^4 to be stable, but deuterium remains unstable. [\[pg.109\]](#)
- Neutron & Proton collisions w/ electrons, positrons, neutrinos, & antineutrinos have nearly ended. [\[pg.109\]](#)
- Free neutron decay beginning to become important; 10% of remaining Neutrons now decay into Protons every 100 seconds – new balance is now 14% neutrons and 86% protons. [\[pg.109\]](#)

TIME = ~3 minutes 46 seconds [based on approximation of 10^9 photons per nucleon]

CHARACTERISTIC EXPANSION TIME = ???? seconds

TEMP = 0.9×10^9 Kelvin (for a nuclear particle:photon ratio of 1:10⁹)

CIRCUMFERENCE = prop. To T -> = ($\sim 92 \times 10^9$ LY)x(2.72K/ 0.9×10^9 K) = 278 LY

- Temperature drops to a point (exact T uncertain as that is based on an exact nuclear particle:photon ratio) where deuterium nuclei remain stable, allowing heavier nuclei to be built up rapidly by the two-particle reactions above, but there is a second bottleneck: nuclei heavier than helium cannot form as there are no stable nuclei with five or eight nuclear particles. Thus, nearly all the remaining neutrons are cooked into helium nuclei. [pg.109-110]
- For a ratio of 10^9 photons per nucleon, nucleosynthesis will begin at a temperature of 0.9×10^9 K and neutron decay would have shifted to a Neutron – Proton balance of 13% neutrons, 87% protons. [pg.110]
- “After nucleosynthesis, the fraction by weight of helium is just equal to the fraction of all nuclear particles that are bound into helium; half of these are neutrons, and essentially all neutrons are bound into helium, so the fraction by weight of helium is simply twice the fraction of neutrons among nuclear particles, or about 26%.” (Note: if the nucleon density is a little higher at this time, fewer neutrons would have decayed - allowing nucleosynthesis to proceed a bit earlier - generating more helium, but likely no more than 28% by weight. [pg.110]

I am confused by this last quote.

TIME = ~34 minutes 40 seconds

CHARACTERISTIC EXPANSION TIME = 1.25 hours

TEMP = 3×10^8 Kelvin

CIRCUMFRENCE = prop. to T -> = ($\sim 92 \times 10^9$ LY)x(2.72K/ 3×10^8 K) = 834 LY

- Electrons and positrons are completely annihilated except for 1 part in 10^9 – that excess is needed to balance the proton charges. The annihilation has given photons a 40.1% higher temperature than the neutrinos. (see note 6, pg. 178). [pg.111]
- The energy density is equivalent to a mass density of 9.9% that of water and consists of 31% neutrinos and antineutrinos, and 69% photons – yielding a characteristic expansion time of ~1.25 hours. [pg.111]
- Nuclear processes have stopped and nucleons are bound into helium nuclei (22-28% by weight), or are free protons (72-78% by weight). [pg.111]
- There is one electron for each free or bound proton; still too hot for stable atoms to form. [pg.111]

Continued expansion and cooling through 700,000 years.

CHARACTERISTIC EXPANSION TIME = ?????

TEMP = falls from 10^8 Kelvin to 2.72K

CIRCUMFRENCE = prop. to T -> = ($\sim 92 \times 10^9$ LY)x(2.72K/ 3000 K) = 83×10^6 LY

- Electrons and nuclei able to form stable atoms. The taking up of free electrons allow photons to have essentially limitless mean free paths – decoupling radiation from matter and allowing matter to begin gravitationally clumping into galaxies and stars. [pg.112]
- Studies have shown that the sun and other stars begin with 20-30% helium and 70-80% hydrogen. Calculating cosmological helium production from the ~3K background temperature yields the same values. [pg.113]
- Using the background radiation temperature, if we could determine the primordial deuterium present before stellar cooking began, we could determine the precise photon:nucleon ratio (based on Wagoner’s work): [pg.114]

- **Photons:nucleons** **Deuterium PPM**

100 x 10 ⁶	0.00008
1000 x 10 ⁶	16
10,000 x 10 ⁶	600
- Spectroscopic study of the sun indicates its surface contains <4PPM deuterium, but most of the surface deuterium would have fused with hydrogen into He³. [\[pg.115\]](#)
- Satellite based spectral analysis of the interstellar medium indicate deuterium at ~20PPM – equivalent to a primordial photon:nucleon ratio of ~1.1 x 10⁹ photons:1 nucleon. [\[pg.115-116\]](#)
- Assuming deuterium at 20PPM, and at the CMB temp. of ~3K, there are 550,000 photons/10⁶ liters and therefore ~500 nucleons/10⁶ liters, which is much less than a closed universe which requires ~3000 nucleons/10⁶ liters. **To close the universe, the deuterium abundance should be less than we have found it to be to date. It is noted that stellar processes could have produced much of the deuterium we see, but then we should see more lithium, beryllium, and boron...** [\[pg.116\]](#)
- The wavelengths of neutrinos, that exited thermal equilibrium when the temperature fell to ~10¹⁰K, continued expanding proportionally to the size of the universe with their energy distributions (black body spectrum) remaining fixed – shifting to those longer wavelengths. Their numbers have also remained fixed. The present neutrino temperature therefore remains approximately the same as the photon background and their numbers are expected to be in a ratio with nucleons of ~ 10⁹:1. [\[pg.117-118\]](#)
- The neutrino black-body curve is expected to be shifted to slightly lower temperatures as it was after their exit from thermal equilibrium that electrons and positrons began to annihilate, thereby heating the photon spectrum then obtaining, but not heating the neutrinos present. Thus, neutrino temperatures are expected to be a little less than the photon temperatures at that time and that difference should have persisted to the present. Neutrino temperature is calculated to be < photon temperatures by a factor of the cube root of 4/11, or 71.38%; thus, the neutrinos and antineutrinos contribute 45.42% as much energy to the universe as photons (see mathematical note 6, pg. 178). Therefore, the neutrino background black-body spectrum should peak at 71.38% of the photon background, or approximately 2K. (This assumes the lepton number density is small and the number ratio of neutrinos to antineutrinos should be 1 part in 10⁹. [\[pg.118\]](#)
- If the lepton number density is comparable to the photon number density, then “there would be a ‘degeneracy,’ an appreciable excess of neutrinos (or antineutrinos) and a deficiency of antineutrinos (or neutrinos). Such a degeneracy would affect the shifting neutron-proton balance in the first three minutes, and hence would change the amounts of helium and deuterium produced cosmologically. **Observation of the 2K cosmic neutrino and antineutrino background would immediately settle the question of whether the universe has a large lepton number, but much more important, it would prove that the standard model of the early universe is really true.”** [\[pg.119\]](#)
- It must be kept in mind that we do not have proof that the universe is in fact was homogeneous and isotropic prior to the CMB-LSS. There may have been more disorder sometime beforehand that was smoothed out by some mechanism. [\[pg.119-120\]](#)

6. A HISTORICAL DIVERSION [\[pg.122-132\]](#) [\[pg. 114-123 1977\]](#)

Why was the CMB discovered accidentally, and why was its detection not sought earlier – despite its earlier prediction?

- Note: the measured present CMB value and mass density of the universe allows calculation of light element cosmic abundances that agree with observations. [\[pg.122\]](#)
- By measuring present cosmic levels of 20-30% helium, and 70-80% hydrogen, it could have been inferred that nucleosynthesis must have begun when the neutron fraction of total nuclear particles had dropped to 10-15%, recalling that the **present helium abundance by weight is just twice the neutron fraction at the time of nucleosynthesis. This value of 10-15% was reached when the temperature = ~10⁹K. This would**

have led to roughly estimate the density of nuclear particles at that temperature. In addition, the density of photons at that temperature can be calculated from the properties of black-body radiation – thus allowing the ratio of the numbers of photons:nucleons to be known; this ratio would continue to the present. From observations of the present density of nuclear particles, one could predict the present density of photons and infer the existence of a CMB with temperature between 1 – 10K. This data was available during the 1940s-1950s. [\[pg.123\]](#)

What is the explanation for the statement “the present helium abundance by weight is just twice the neutron fraction at the time of nucleosynthesis.” [\[pg.123\]](#)

- In the late 1940s, a “big bang” cosmological model was proffered by George Gamow, Ralph A. Alpher and Robert Herman. They assumed the universe began as pure neutrons that decayed into a proportion of protons, electrons, and antineutrinos over time, which would subsequently synthesize nuclei by a rapid sequence of neutron captures. Alpher and Herman found that this process could lead to the currently observed abundances if the ratio of photons to nucleons was set to 10⁹:1. **Using estimates of the present cosmic density of nucleons led them to predict a CMB of a present temperature of 5K.** [Note – their assumption of a start of pure Neutrons is incorrect; there were actually equal numbers of Neutrons and Protons and the conversions between them was secondary to collisions with electrons, positrons, neutrinos, and antineutrinos, not through Neutron radioactive decay. This was noted in 1950 by C. Hayashi, and in 1953 Alpher and Herman (and J. W. Follin, Jr) revised the model and carried out correct calculations of the shifting neutron-proton balance. Nonetheless, no one undertook a search for a CMB at that time. [\[pg.123-124\]](#)
- Rather it was only in 1964 that Ya. B. Zeldovich in Russia and Hoyle and R. J. Tayler in England, and Peebles in the US independently picked up that work. None of this was known to Penzias and Wilson who found the CMB in 1965 without initially understanding its significance. [\[pg. 125\]](#)
- That most of the universe’s mass is in the form of hydrogen has been known since the 1950s. This is sufficient information to know that a large ratio of photons:nucleons exist, otherwise all the hydrogen would have cooked into helium and heavier elements in the early universe. [\[pg. 126\]](#)
- The 3K CMB measurement could have been made by the mid 1940s (wartime radar research), but this did not occur. There are several counter examples of large experimental machines that were built to detect new particles including neutrons, antiprotons, and neutrinos. [\[pg. 126\]](#)
- **How is it that radio astronomers did not know of the nucleosynthesis work and so did not know of a need to search for a CMB from the 1940s through the early 1960s? What went wrong?** [\[pg. 127\]](#)

FIRST

- Gamow, Alpher, Herman, and Follin, et al., generated a big bang theory of all complex nuclei – built in the early universe via rapid addition of neutrons but could not explain why there were no heavy elements formed – making it difficult for scientists to take the Big-Bang nucleosynthesis model seriously. (**There are no stable nuclei with five or eight nucleons – it is not possible to build nuclei heavier than helium by adding neutrons or protons to helium nuclei or by fusing pairs of helium nuclei – first noted by Enrico Fermi and Anthony Turkevich**).
- In 1952 E. E. Salpeter showed that the five or eight nucleon stability gap could be overcome in stellar cores where two helium nuclei could fuse into beryllium-8, which could rapidly fuse with another helium nucleus and form stable carbon-12. In 1957 Geoffrey and Margaret Burbidge, Fowler, and Hoyle showed that heavy elements could be synthesized in stars and via stellar explosions during intense neutron fluxes would obtain. **By 1940 Hans Bethe and others made clear that the key process in stellar fusion was of combining four protons into a helium nucleus. This caused many astrophysicists to strongly hold that stars were the source of all nuclei beyond hydrogen.** [\[pg. 128\]](#)

What is the explanation for there being no stable nuclei with 5 or 8 nucleons? [\[pg. 128\]](#)

- However, it was not immediately understood that the stellar theory of nucleosynthesis could not however build up the 25-30% helium abundance seen – that degree of fusion would release more energy than stars release throughout their lifetimes. [pg. 129]
- “The cosmological theory gets rid of this energy very nicely – it is simply lost in the general red shift.” [pg. 129]
- **In 1964 Hoyle and R. J. Tayler pointed out that the large helium abundance present in the current universe could not have been created in stars – rather they calculated that helium produced in the early stages of a big bang could reach a current abundance of 36% by weight.** In order to obtain this value, they made an arbitrary time of nucleosynthesis set to 5×10^9 K (despite the fact that this assumption depends on the value chosen for a then unknown parameter, the ratio of photons to nuclear particles. “Had they used their calculation to estimate this ratio from the observed helium abundance, they could have predicted a present CMB” within an order of magnitude to that later discovered. [pg. 129-130]

SECOND

- There was a breakdown in communication between theorists and experimentalists. Radio astronomy was still in its infancy during the 1940s – 1960s, and most theorists did not realize a 3K CMB could ever be detected. [pg. 130]
- Interestingly, in 1964 Ya. B. Zeldovich generated a review article in which he correctly calculated cosmic helium abundance for two possible values of the present radiation temperature and correctly emphasized that these quantities are related because the number of photons per nuclear particle (or the entropy per nuclear particle) does not change with time. He was apparently misled by the use of the term “sky temperature” used in a 1961 article by E. A. Ohm in the *Bell System Technical Journal* to conclude that the radiation temperature had been measured to be < 1K. This and a low estimate of cosmic helium abundance, led Zeldovich to abandon the idea of a hot early universe. [Interesting note: The antenna used by Ohm was the same 20-foot horn reflector later used by Penzias and Wilson.] [pg. 130-131]
- Information between theorists and experimentalists was not crossing well (Penzias and Wilson were unfamiliar with the Alpher-Herman predictions as they worked on their antenna). [pg. 131]

THIRD

- “...it was extraordinarily difficult for physicists to take seriously any theory of the early universe.” The idea that data and thinking could discover information about the first minutes of the universe seemed too remote. “...our mistake is not that we take our theories too seriously, but that we do not take them seriously enough. It is always hard to realize that these numbers and equations we play with at our desks have something to do with the real world. Even worse, there often seems to be a general agreement that certain phenomena are just not fit subjects for respectable theoretical and experimental effort. [pg. 131]
- “The most important thing accomplished by the ultimate discovery of the 3K radiation background in 1965 was to force us all to take seriously the idea that there *was* an early universe.” [pg. 132]

6. THE FIRST ONE-HUNDREDTH SECOND [pg.133-149] [pg. 124-139 1977]

- 10^{11} K is the temperature of the strong force interactions at the extremely short range of 10^{-13} cm. At such a range, two protons experience strong force binding them 100-times that of the repulsive force of their electric charges. This is why the strong force is able to bind together nearly 100 protons within the largest nuclei. [pg. 134-135]
- In a thermonuclear explosion, the energy released is the result of a rearrangement of neutrons and protons allowing a more tightly bound arrangement. [pg. 135]

- **Electromagnetic force interactions:** When two electrons scatter off on another, the rates of Feynman Diagram reactions of each of the emissions and absorption of photons and electron-positron pairs are calculated by squaring of the sum of each of the contributing reactions. Adding one more internal line of interaction in any diagram lowers the contribution of the diagram by a factor roughly equal to the fine structure constant (1/137.036). **Thus, complex diagrams (actually infinite) yield only small contributions – allowing them to be essentially ignored** - instead adding up the contributions of only a few simple diagrams. [\[pg. 135\]](#)
- **Strong force hadronic interactions:** In calculating a full interaction, each component contributes by a factor of 1, as opposed to the EM force factor of 1/137. Thus, in complex diagrams – each contribution adds as much to the whole as every other, making calculations extremely difficult. [\[pg. 135-136\]](#)
- Strong interactions involve hadrons only. The difference in strength of the EM vs. the strong force yields atomic electron clouds being 10^5 times larger than the size of atomic nuclei, and causes chemical forces holding atoms together in molecules 10^6 times weaker than the forces holding neutrons and protons together in nuclei. [\[pg. 137\]](#)
- 10^{11}K is below the threshold temperature for all hadrons (*the lightest hadron is the pi-meson with a threshold temperature of 1.6×10^{12}*). Thus, at 10^{11}K the only particles present in large numbers were leptons and photons, and interactions between them can be ignored. [\[pg. 137\]](#)
- At temperatures $> 1.6 \times 10^{12}\text{K}$, hadrons and antihadrons were present in large numbers.
 - **TWO THEORETICAL VIEWS**
 - **(1) “Nuclear Democracy”:** Each hadron can be thought of as fundamental and possibly unlimited in species number. As reaction energies rise, more hadrons are created as are new species of them, rather than causing temperatures to rise. Thus, with increasing energy, the temperature does not rise as fast as it would if the numbers of hadron species were fixed. There may even be a maximum possible temperature at which energy density becomes infinite. This idea was originally offered by R. Hagedorn of CERN, and further developed by Kerson Huang of M.I.T. and Steven Weinberg. That maximum temperature is estimated to be about $2 \times 10^{12}\text{K}$ which would exist at ~ 0.01 second “before the first frame of Chapter V [begun at 0.01 second].” [\[pg. 137-139\]](#)
 - **(2) “Not all particles are equal”:** hadrons are composites of quarks, as first described by Murray Gell-Mann and independently by George Zweig, both of Cal Tech. In this model, quarks are fundamental and – as was discovered at SLAC – are confined to maximum distances from one another with the force between them vanishing when they are very close together. This suggests that at very high temperatures “several” $\times 10^{12}\text{K}$ hadrons would break up into their constituent quarks. At those temperatures the universe would consist of photons, leptons, antileptons, quarks, and antiquarks all moving as free particles and each particle species therefore in effect furnishing just one more kind of black-body radiation. And a true beginning would have occurred at ~ 0.01 second “before the first frame of Chapter V [begun at 0.01 second] at an infinite temperature and infinite density. Events at these high densities and temperatures is described in “non-Abelian gauge theories” (1973 – Hugh David Politzer of Harvard, David Gross and Frank Wilczek of Princeton) that allow for asymptotically short distances and high energies where quarks act as free particles (asymptotic freedom). [\[pg. 139-141\]](#)
- “...it has so far proved impossible to break up any hadron into its constituent quarks.” (as written in 1977.) Asymptotic freedom of quarks at short range is consistent with the energy required to separate quarks is sufficient to create a pairing quark at that extreme energy – thus no true free and independent quarks are possible. [\[pg. 141-142\]](#)
- **WEAK PHASE TRANSITION:** The weak interaction is $\sim 10^{-7}$ times weaker than the electromagnetic interaction. The electroweak field theory was introduced in 1967 by Steven Weinberg and independently by Abdus Salam in 1968. The theory predicted a class of weak interaction – referred to as neutral currents; this

was experimentally confirmed in 1973. This theory was also within the mathematics of non-abelian gauge theories. [pg. 142-143]

- Gauge theories may provide a unified basis for understanding all the forces of nature. "This view is supported by a property of the unified gauge theories that had been conjectured by Salam and myself [the author], and was proved in 1971 by Gerard 't Hooft and Benjamih Lee: the contribution of complicated Feynman diagrams, though apparently infinite, gives finite results for the rates of all physical processes." [pg. 143]
- This **electroweak gauge theory exhibits a phase transition at $\sim 3 \times 10^{15}$ K** below which the weak interactions were weak and of short range. Above that temp., the weak force obeyed the same inverse-square law as is present in electromagnetic interactions and at about the same strength. Below that critical temperature, the symmetry between the weak and electromagnetic forces was broken. [pg. 144]
- Of interest, phase transitions need not be uniform throughout the space of the transition – there may be different broken symmetries than observed within our particular domain. [pg. 144]
- Gravity has not yet been shown to have any effect on the very early universe given the weakness of the interaction: **the gravitational force between the electron and proton in a hydrogen atom is weaker than the electric force by 10^{39}** . [pg. 145-146]
- **At 10^{-24} second the gravitational force tidal effects would have been sufficient to produce particle-antiparticle pairs**, but at that time the temperature would have been enormous as would have been particles in thermal equilibrium already.
- Nonetheless, a time can be calculated (**10^{-43} second**) along with a corresponding temperature (10^{32} K) when gravity would have been as strong as the strong force. At such a time **the "horizon" would be closer than one wavelength of a typical particle in thermal equilibrium... i.e., each particle would be about as big as the observable universe.** [pg. 145-146]
- The thermal equilibrium that was present before one-second was broken at that time. With one exception – gravitational waves - "As far as we know, nothing that we can observe depends on the history of the universe prior to that time." [pg. 146-147]

"It is as if a dinner were prepared with great care – the freshest ingredients, the most carefully chosen spices, the finest wines – and then thrown all together in a great pot to boil for a few hours. It would be difficult for even the most discriminating diner to know what he was being served." [pg. 147]

- However, gravitational waves – traveling at the speed of light – should exist, [but had not yet been discovered as of the writing of the book in 1977 or the revision in 1993]. [pg. 148]
- Gravitational radiation would have gone out of thermal equilibrium with the other contents of the universe at a very early time at $\sim 10^{32}$ K. "Since then, the effective temperature of the gravitational radiation has simply dropped in inverse proportion to the size of the universe." [pg. 148] Note that the annihilation of quark – antiquark and lepton-antilepton pairs headed the universe after gravitational radiation fell out of thermal equilibrium. Thus, gravitational radiation temperature would be expected to be less than that of photons or neutrinos, perhaps on the order of 1K. [pg. 148]
- One possibility – never was a state of infinite density, or maybe there was a beginning when cause and effect (time) had no meaning. [pg. 149]

8. EPILOGUE: THE PROSPECT AHEAD [pg.150-155] [pg. 140-144 1977]

- If the universe is open, or if it is exactly balanced, it will gradually cool further – all stars will cease their thermonuclear processes. This depends on whether the cosmic density is $>$, $<$, or equal to the critical density. [pg.150-151]

- If the universe is closed, it will eventually contract and the background radiation will rise to beyond our present room temperature (300K) and beyond and atoms and then other composite particles will dissociate – leading to a cosmic soup of individual particles and radiations and finally to a state of infinite energy density and temperature where time is undefined. If the density is twice the critical value, maximum dilatation will be just twice as large as present, and the CMB will fall to about 1.5K before contraction occurs. When the universe re-contracted to $1/100^{\text{th}}$ the present size the CMB would result in the night sky temperature being equal to our day sky now at 300K. After millions of years all would dissolve in a cosmic soup of subatomic particles... [\[pg.151-152\]](#)
- Another possibility for a closed universe is that some unknown process could result in a “bounce” at some point in energy density and time. However, with each cycle the ratio of photons to nuclear particles would be slightly increased by a “kind of friction (known as “bulk viscosity”) as the universe expands and contracts. As far as we know, the universe would then start each new cycle with a new, slightly larger ratio of photons to nuclear particles. Right now, this ratio is large, but not infinite., so it is hard to see how the universe could have previously experienced an infinite number of cycles.” [\[pg.151-155\]](#)

“The effort to understand the universe is one of the very few things that lifts human life a little above the level of farce, and gives it some of the grace of tragedy.” [\[pg.155\]](#)

AFTERWORD: COSMOLOGY SINCE 1977 (1993) [\[pg.179-191\]](#)

- **H_0 :** measurement uncertainties continue [\[pg.179-180\]](#)
- **CMB:** 1989 COBE 2.735K black-body spectrum w/ anisotropies on the order of 30×10^{-6} K on angular scales of 7-180 deg. To observe the beginning of galaxies, need to explore angles <7 degrees. [\[pg.180-182\]](#)
- **DARK MATTER:** Observations of spiral galaxy rotations do not show a velocity drop off of orbiting stars far from the galactic centers. Galactic motion analysis suggests baryonic mass total of 10-40% of that required for a flat universe. The amounts and ratios of light elements during primordial nucleosynthesis was dependent on the ratio of atomic particles to photons. [\[pg.182\]](#) “A relatively high ratio of atomic particles to photons would allow the nuclear reactions that convert hydrogen to helium to proceed more nearly to completion, reducing the amount of matter left over in the form of less tightly bound light elements like deuterium or lithium-7. Data available by 1993 on Li-7 primordial abundance indicates baryonic matter = ~3% of that required for a flat universe. [\[pg.182\]](#)
- **LIGHT ELEMENT ABUNDANCE:** The amounts of light elements is influenced by the numbers of neutrino species... the more types would drive a faster expansion and a greater proportion of helium. CERN experiments in 1990 on Z^0 decay conclusively showed there are three neutrino species. With the single free parameter of nucleons:photons, the present abundances of hydrogen, helium-4, deuterium, helium-3, and lithium-7 can be deduced! [\[pg.182-183\]](#)
- **MATTER-ANTIMATTER ASYMMETRY:** experiments in 1964 demonstrated there are various mechanisms that violate the strict conservation of baryon and lepton numbers. The ratio of atomic particles to photons is presently $1:10^9 - 1:10^{10}$. This asymmetry may have occurred during electroweak symmetry breaking at 10^{16} K. [\[pg.183-184\]](#)
- **CRITICAL DENSITY AND THE ANTHROPIC PRINCIPLE:** “Many astronomers and physicists have suspected for decades that the mass density of the universe is precisely at the critical value. The argument is essentially aesthetic. As the universe expands, the ratio of its mass density to the critical value changes with time; in all cases it starts near 100%, decreasing if initially less than 100%, and increasing if initially greater than 100%. Yet now, billions of years after the ‘big bang,’ the measured mass density it’s still within a factor of 10 of the

critical value. This is only possible if the mass density near the beginning (say, in the first few seconds) was incredibly close to the critical value. It seems hard to understand why the mass density should have had such a value, unless it always has been precisely at the critical value.” [\[pg.184-185\]](#)

“One way to tell whether the universe has a critical mass density is to measure the rate at which the expansion of the universe is slowing down. We can do this in principle in the same way we measure the Hubble constant, observing how velocities of distant galaxies increase with distance. The problem here is the same as it has been for over half a century: the deceleration of the cosmic expansion can only be measured by studying galaxies so distant that the rate of cosmic expansion has decreased appreciably since the light we see was emitted. But since we are seeing these very distant galaxies as they were so long ago, their intrinsic luminosities may have been very different from what we would infer from our studies of nearer galaxies. Thus, we cannot use their apparent luminosities to a further distance. It may be, however, that the physical size of galaxies evolves less than their luminosities, so that observation of apparent sizes gives a more reliable measure of distance than apparent luminosities. A 1992 survey of this sort indicated that the universe’s expansion is slowing down at close to the rate that would be expected if the universe does contain a critical mass density.” [\[pg.185\]](#)

“If the mass of the universe is indeed at the critical value it could not be in the form of ordinary matter without wrecking the agreement between calculations of the production of light elements and the first few minutes and the observation of the present abundance of these elements. Indeed, whether or not the mass of the universe is at the critical value, it is probably greater than the maximum value in ordinary matter allowed by calculations of ‘big bang’ nucleosynthesis. So, of what does the mass of the universe consist? In the 1970s and 1980s, there was wide speculation that the missing mass is contained in neutrinos that are very light but not quite massless. As discussed in Chapter IV, neutrinos are about as abundant today as photons, and it is easily calculated that if neutrinos have masses of about 20 electron volts (that is, about 40 millionths the mass of an electron), they would provide the whole critical mass. But recent experiments on nuclear beta decay suggest that neutrino masses are much smaller, if not zero.” [\[pg.185-186\]](#)

“The missing mass is also possibly contained in particles that are much heavier but also much less abundant than these supposed 20-electron-volt neutrinos. Any type of particle, however heavy, would have been abundant, along with the corresponding anti-particle, at early times when the temperature was very high. As the universe expanded and cooled, the heaviest particles and anti-particles would have annihilated, until they became so rare that they could no longer find each other to annihilate. If stable, the remaining unannihilated particles and anti-particles would still be present today. By knowing the mass of any sort of particle and its annihilation rate with its anti-particle, we can calculate how many of these particles and antiparticles would be present today and how much they would contribute to the present mass of the universe. In recent years particle physicist have speculated about a variety of heavy particles of this sort. Presently, the most attractive possibility is that the missing mass consists of stable particles, known as photino’s or neutralinos, with masses in the range of 10 to 10,000 proton masses and with slow annihilation rates, that would be required by a hypothesized symmetry of elementary particles called supersymmetry. There are experiments in progress to search for these particles by looking for effects of their collision with the atoms in sensitive detectors. It seems also likely that such exotic heavy particles, if they exist, could be produced at a sufficiently powerful new accelerator, such as the SSC or LHC. Finding these particles at the SSC or the LHC would mark a revolution in cosmology as well as in elementary particle physics.” [\[pg.185-186\]](#)

“I should mention another popular candidate for the missing mass – a particle known as the axion, hypothesized in 1977 to solve some problems of particle physics. These particles would have been left over from the ‘big bang’ in numbers vastly larger than those for photons or neutrinos and would provide a critical mass density if their mass were roughly of the order of a hundred-thousandth of an electron volt. Experimental us are planning searches for cosmic axions, but so far there is no experimental evidence that they actually exist.” [\[pg. 186\]](#)

“There is yet another candidate for the missing mass, and it involves a property of empty space itself. In any sort of quantum theory of fields, the vacuum receives an enormous energy from continual quantum fluctuations in electromagnetic and other fields. According to general relativity, this vacuum energy would

produce a gravitational field equivalent to that produced by a mass density spread uniformly throughout the empty space. We cannot actually calculate this vacuum mass density, because our calculations show that the largest contributions come from fluctuations of a size so very small that at the scales of distance our present theory of gravitation becomes unreliable. If we arbitrarily include only fluctuations of size large enough for us to rely on known theories, then we find a vacuum mass density larger in the greatest value allowed by observations of the expansion of the universe (which is roughly two or three times the critical value). This density would be larger by a factor of 120 orders of magnitude. If we were to take this calculation seriously, it would undoubtedly be the most impressive quantitative disagreement between theory and experiment in the history of science!" [\[pg. 186\]](#)

"The vacuum mass density produced by quantum fluctuations acts in the same way as the cosmological constant term (discussed in Chapter II) introduced by Einstein into his field equations in 1917. Einstein had been trying to construct a static cosmological model, and later came to regret the introduction of the cosmological constant when it became clear that the universe is expanding, yet the term remained a logical possibility. In fact, the cosmological constant is the only term that can be added to the gravitational field equations without violating Einstein's underlying assumption about the equivalence of all coordinate systems (aside from terms that become unimportant at cosmological distances). To say that a cosmological constant term is unnecessary is not enough; our experiences in quantum field theory over the past half-century indicates that any term in the field equations that is not forbidden by some fundamental principle is likely to be present." [\[pg. 186\]](#)

"The problem of the enormous vacuum mass density and the issue of whether or not to include the cosmological constant in the field equations may answer each other. That is, there may be a cosmological constant in the field equations whose value just cancels the effects of the vacuum mass density produced by quantum fluctuations. But to avoid conflict with astronomical observation, this cancellation would have to be accurate to at least 120 decimal places. Why in the world should the cosmological constant be so precisely fine-tuned?" [\[pg. 186-187\]](#)

"Theoretical physicists have been grappling with this question for decades, without much success. Some constants of nature are fixed by fundamental principles in terms of other constants. One example is the Rydberg constant, which gives the energies of the various states of a hydrogen atom, and can be calculated in terms of the mass and charge of the electron and the Planck constant of quantum mechanics. But **no one knows any principle that fixes the cosmological constant**. In 1983 in 1984, excitement surged over the possibility that the problem of the cosmological constant and vacuum mass density might be solved within the context of quantum cosmology. Calculation showed that the universe is probably not in a state with definite values for whatever constants of nature are not fixed by fundamental principles, such as (perhaps) the cosmological constant. Rather, the universe seems to be described by a quantum mechanical wave function containing many terms, each with a different set of values for these constants. **As soon as humans (or anyone else) start making measurements, they find themselves in a state with definite values for the constants of nature, but it is impossible to predict what values they find, only the probabilities. Early calculations indicated that these probabilities sharply peak at a value of the cosmological constant that would just cancel the vacuum energy density once the universe becomes sufficiently large and cool.** This result has been challenged, however, and the issue will probably not be settled until we have a better understanding of how to apply quantum mechanics to the whole universe." [\[pg. 187\]](#)

"This episode has left us with a useful lesson. Even at the probability distribution for constants like the cosmological constant does not have any sharp peaks, it is not unreasonable to suppose that there is some probability distribution that governs the likelihood of finding specific values for these constants. Whatever the shape of this distribution, **there is only a limited range of values for these constants that could possibly be found by any intelligent observer, since there is only a limited range of values that allows for the appearance and evolution of life and intelligence. This idea – that the constants of nature must take values that allow for the existence of life and intelligence – is known as the anthropic principle.** Though this principle has not been popular among scientists, quantum cosmology provides a context in which it

becomes simple common sense. Anthropic reasoning could also be justified if the universe passes through phases, or contains distant regions in which the “constants” of nature take different values.” [pg. 187]

“Such anthropic arguments refer not to the vacuum mass density itself or the cosmological constant itself, but only to the net vacuum mass density, which includes the equivalent contribution of the cosmological constant. It is the net vacuum mass density that serves as a source (along with any ordinary matter) of the cosmic gravitational field. Specifically, if the net cosmic mass density were very much larger than the present critical mass density and were *negative*, then the universe would run through its cycle of expansion and contraction so rapidly that there would be no time for stars to form, much less life for intelligence. If the net vacuum mass density were very much larger than the present critical mass density and were *positive*, then the expansion of the universe would continue forever. Any clumps of matter that formed in the early universe, however, would be ripped apart by a long-range repulsive force, and, without galaxies or stars, there would be no place for life to arise. The anthropic principle could thus explain why the net vacuum mass density is not much larger than the present critical density.” [pg. 187-188]

“The really intriguing thing about this line of reasoning is that the anthropic principle, if valid, would not require the net vacuum mass density to vanish, or even to be smaller than the present critical density. **We know (from the red shifts of distant quasars) that gravitational clumps had already begun to form when the universe was 6-times smaller than its present size. At that time, the density of ordinary matter was 6^3 , or 216, times larger than at present; a net vacuum mass density would thus have had no effect on the formation of gravitational condensations unless it were at least ~ 100 -times larger than the present cosmic density of ordinary matter.** A smaller vacuum mass density could have interfered with the formation of galaxies at later times, but a net vacuum mass density ~ 10 -20-times the present mass density of ordinary matter would have left plenty of time for galaxy formation. The anthropic principle, therefore, provides no reason why a positive net vacuum mass density should be smaller than about 10- to 20-times the present mass density of matter (including whatever dark matter is present in galaxies and clusters of galaxies). Is it possible that 80 or 90 percent of the critical mass arises from the vacuum, with the remainder made up of ordinary matter (mostly dark) of one sort or another?” [pg. 188]

“Fortunately, this is a question that can be settled by astronomical observation. There is a crucial difference between the mass density of ordinary matter, on the one hand, and that produced by quantum vacuum fluctuations and/or a cosmological constant: the density of ordinary matter has been steadily decreasing as the universe has expanded while the vacuum mass density has been constant. This makes for large differences and what we see when we look out to very large distances, differences that can be used to discriminate between a critical density made up of ordinary matter, or are rising from a net vacuum mass density.” [pg. 188]

This topic: “net vacuum mass density” is confusing. Is “net vacuum mass density” what we refer to as “dark energy”? Weinberg’s page 188 paragraph beginning: “The really intriguing thing...” can hopefully be clarified. Also, the following paragraph beginning: “Fortunately, this is a question that can be settled by astronomical observation” can also be discussed.

“One point in favor of a large vacuum mass density is that it would help resolve a potential conflict between measurements of the Hubble constant and the ages of stars. In a universe with a critical density made up of ordinary matter, the age of the universe is inversely proportional to the Hubble constant: about 8-billion years for Hubble constant of 80km/sec/megaparsec, and 16-billion years for a Hubble constant of 40km/sec/megaparsec. But comparisons of the observed colors and luminosities of stars in globular clusters with computer calculations of stellar evolution indicates that these stars are between 12- and 18-billion years old. Also, studies of the abundance of various radioactive isotopes show that our galaxy is at least 10-billion years old. If it turns out that the Hubble constant is near the high-end of the currently quoted range, then we will face the paradox of the universe younger than its oldest stars. But if we suppose instead that the mass density of the universe arises mostly from a vacuum mass density, then its density in the past would have been lower. As a result, the expansion would have been slower, and for any given Hubble constant the universe would be older – sufficiently older to remove the conflict with the ages of the old

stars.” [pg. 188-189]

“A large vacuum mass density would also affect counts of galaxies at various redshifts or apparent luminosities; counts of galaxies that act as gravitational lenses (galaxies whose gravitational field focuses the light of more distant objects along the same line of sight); in the variation of apparent sizes of galaxies with redshifts. So far, the evidence seems to be against a large vacuum contribution to the cosmic mass density, but it is too early to be sure. If the net vacuum mass density is confirmed to be really much smaller than the present density of ordinary matter, then an anthropic explanation of the value of the cosmological constant will become untenable: there is no anthropic reason why the net vacuum mass density should be that small.” [pg. 189]

Again, the concept of net vacuum mass density is not clear. “If the net vacuum mass density is confirmed to be really much smaller than the present density of ordinary matter, then an anthropic explanation of the value of the cosmological constant will become untenable: there is no anthropic reason why the net vacuum mass density should be that small.”

- **INFLATION:** “Whatever the net vacuum mass density may be in the present epoch of our expanding universe, there are strong reasons to believe in an earlier epoch when the net vacuum mass density was very large. This is because (as discussed in Chapter VII) the universe has expanded and cooled through a series of cosmic phase transitions, like the freezing of water when the temperature drops below 0°C. **In these transitions, various fields that permeate “empty” space suddenly shifted their values, with a consequence shift in the energy density, and equivalent mass density of the vacuum. If the fields do not immediately reach their equilibrium values, then the vacuum will have an excess energy density that will drive a rapid expansion of the universe.”** [pg. 189]

“Theorists became intensely interested in such phase transitions in the early 1980s, when it was pointed out that this rapid expansion, known as ‘inflation,’ would solve a number of outstanding cosmological problems. For one thing, it was known since the late 1970s that early phase transitions would have produced large numbers of isolated magnetic poles, in contradiction to observe upper limits on the number of these ‘monopoles’ present in the universe today. Inflation would dilute the number of monopole’s safely below the observational limits. More important, inflationary cosmologies also resolve (or at least mitigate) a paradox arising from the observed uniformity of the microwave radiation background. Any two light rays they come to us from points in the sky more than about 2-degrees apart must have been emitted from sources so far apart when the universe was a million years old that there would not have been time for any signal to have traveled at less than the speed of light from one light source to the other. But then, what physical mechanism could have produced be observed near equality of the microwave radiation intensity in all directions? How could we explain the fact the microwave radiation temperature is nearly uniform at angular scales larger than 7-degrees – so much that it is only recently, we observations from the COBE satellite, that we have found any departures from uniformity? In inflationary cosmologies there was plenty of time during the early inflationary epoch for physical processes to smooth out the distribution of matter and energy and to produce the observed high degree of uniformity of the cosmic microwave radiation background.” [pg. 189-190]

“There are by now and number of variants of inflationary cosmology. In one version, inflation is not the result of a delayed phase transition, but rather arises when a localized quantum fluctuation insomnia feel temporarily drives the vacuum energy above its normal value in a small region, which then inflates to enormous size. In this picture, our ‘universe,’ the multi-billion-light-year wide expanding cloud of galaxies that we can see from the earth, is only a sub-universe in a much larger universe that eternally breeds new sub-universes.” [pg. 190]

“Inflationary cosmologies make two characteristic predictions. One is that the mass density must be very close to the critical value. The other is that non-uniformities in the microwave radiation background, which are explained in inflationary cosmologies as quantum fluctuations that have been magnified by inflation, are predicted to have a characteristic ‘flat’ angular distribution of scales larger than 2-degrees. Both of these predictions are in fair agreement with experiment. The cosmic mass density is close enough to the critical

value to make it plausible that the two are equal, and the nonuniformity's in the cosmic microwave background studied by COBE do seem to follow a flat distribution law. Unfortunately, neither of these predictions is unique to inflationary cosmologies, indeed, both were suggested before inflationary cosmologies were developed. It is not clear what sort of astronomical observation will ever be able to confirm the idea of inflation. The impressive progress of observational cosmology since 1977 has done much to strengthen the case for the standard 'big bang' cosmology, but a gap has opened between what theorists are led to speculate and what are astronomers are able to observe." [\[pg. 190\]](#)

Weinberg states in 1977 that "It is not clear what sort of astronomical observation will ever be able to confirm the idea of inflation." Would finding a CMB B-field "curl" signature confirm inflation?

- **WEAK FORCE / VERY HIGH TEMPERATURE PARTICLE PHYSICS:** "Much the same could be said of the recent history of elementary particle physics. The years since 1977 have seen the sequence of brilliant experiments – most dramatically, the discovery from 1983 to 1984 of the W and Z particles that transmit the weak nuclear forces. As a result, serious doubt does not now exist about the correctness of our standard model of the electromagnetic and the weak and strong nuclear forces. In particular, the continued success of the 'asymptotically free' theory of strong interactions has by now made obsolete the speculations in chapter VII about a maximum temperature of 10^{12} K. At higher temperatures, nuclear particles dissolved into the quarks they are made of, and the matter of the universe behaves, quite simply, as a **gas of quarks, leptons, and protons**. The description of matter only becomes greatly difficult at the much higher temperature of 10^{32} K, where gravitation becomes as strong as other forces. Theorists have been speculating about the theory that governs matter at these temperatures, but we are a long way from any direct experimental test of these speculations." [\[pg. 190-191\]](#)
- **STRING THEORIES:** "The most exciting speculative theories studied since 1977 have been the string theories. These replace the description of matter in terms of particles with a description in terms of strings – tiny one-dimensional discontinuities in space-time. The strings can be in any one of an infinite number of modes of vibration, each of which appears to us as a different species of elementary particle. Gravity appears not only naturally but inevitably in string theories; the quantum of gravitational radiation is one of the modes of vibration of a closed string. There may be a maximum temperature in modern string theories, but it would be in the neighborhood of 10^{32} K not 10^{12} K." [\[pg. 191\]](#)

"Unfortunately, there are thousands of versions of string theories, and we do not know how to evaluate their consequences or why one string theory rather than another should describe our universe. But there is one aspect of string theories that is a great potential importance to cosmology. Our familiar four-dimensional space-time continuum is not a truly fundamental ingredient of string theories, but arises in the approximate descriptions of nature that only become valid at temperatures below $\sim 10^{32}$ K. It may be that our real problem will not be to understand the beginning of the universe, or even to decide whether there really was a beginning, but rather to understand nature under conditions, in which time and space have no meaning." [\[pg. 191\]](#)

End.

A Mathematical Supplement

These notes are provided for readers who wish to see some of the mathematics that underlie the non-mathematical exposition presented in the body of this book. It should not be necessary to study these notes in order to follow the discussions in the main part of this book.

Note 1 The Doppler Effect

Suppose that wave crests leave a light source at regular intervals separated by a period T . If the source is moving at a velocity V away from the observer, then during the time between successive crests the source moves a distance VT . This increases the time required for the wave crest to get from the source to the observer by an amount VT/c , where c is the speed of light. Thus, the time between arrival of successive wave crests at the observer is

$$T' = T + \frac{VT}{c}$$

The wavelength of the light upon emission is

$$\lambda = cT$$

and the wavelength when the light arrives is

$$\lambda' = cT'$$

Thus, the ratio of these wavelengths is

$$\lambda'/\lambda = T'/T = 1 + \frac{V}{c}$$

The same reasoning applies if the source is moving toward the observer, except that V is replaced with $-V$. (It also applies for any kind of wave signal, not just light waves.)

For instance, the galaxies of the Virgo cluster are moving away from our galaxy at a speed of about 1,000 kilometers per second. The speed of light is 300,000 kilometers per second. Therefore the wavelength λ' of any spectral line from the Virgo cluster is larger than its normal value λ by a ratio

$$\lambda'/\lambda = 1 + \frac{1,000 \text{ km/sec}}{300,000 \text{ km/sec}} = 1.0033$$

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This is the critical density. (Although this result has been derived here using Newtonian physical principles, it is actually valid even when the contents of the universe are highly relativistic, provided that ρ is interpreted as the total energy density divided by c^2 .)

For instance, if H has the currently popular value of 15 kilometers per second per million light years, then, recalling that there are 9.46×10^{12} kilometers in a light year, we have

$$\rho_c = \frac{3}{8\pi(6.67 \times 10^{-8} \text{ cm}^3/\text{gm sec}^2)} \left(\frac{15 \text{ km/sec}}{10^6 \text{ lt yrs}} \right)^2 = 4.5 \times 10^{-30} \text{ gm/cm}^3$$

There are 6.02×10^{23} nuclear particles per gram, so this value for the present critical density corresponds to about 2.7×10^{-6} nuclear particles per cm^3 , or 0.0027 particles per liter.

Note 3 Expansion Time Scales

Now consider how the parameters of the universe change with time. Suppose that at a time t a typical galaxy of mass m is at a distance $R(t)$ from some arbitrarily chosen central galaxy, say our own. We saw in the last mathematical note that the total (kinetic plus potential) energy of this galaxy is

$$E = mR^2(t) \left[\frac{1}{2}H^2(t) - \frac{4}{3}\pi\rho(t)G \right]$$

where $H(t)$ and $\rho(t)$ are the values of the Hubble "constant" and the cosmic mass density at time t . This must be a true constant. However, we will see below that $\rho(t)$ increases as $R(t) \rightarrow 0$ at least as fast as $1/R^2(t)$, so $\rho(t)R^2(t)$ grows at least as fast as $1/R(t)$ for $R(t)$ going to zero. In order to keep the energy E constant, the two terms in the brackets must therefore nearly cancel, so that for $R(t) \rightarrow 0$ we have

$$\frac{1}{2}H^2(t) \rightarrow \frac{4}{3}\pi\rho(t)G$$

The characteristic expansion time is just the reciprocal of the Hubble constant, or

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Note 2 The Critical Density

Consider a sphere of galaxies of radius R . (For the purposes of this calculation we must take R to be larger than the distance between clusters of galaxies, but smaller than any distance characterizing the universe as a whole.) The mass of this sphere is its volume times the cosmic mass density ρ :

$$M = \frac{4\pi R^3}{3}\rho$$

Newton's theory of gravitation gives the potential energy of any typical galaxy at the surface of this sphere as

$$P.E. = -\frac{mMG}{R} = -\frac{4\pi mR^2\rho G}{3}$$

where m is the mass of the galaxy, and G is Newton's constant of gravitation

$$G = 6.67 \times 10^{-8} \text{ cm}^3/\text{gm sec}^2$$

The velocity of this galaxy is given by the Hubble law as

$$V = HR$$

where H is Hubble's constant. Thus its kinetic energy is given by

$$K.E. = \frac{1}{2}mV^2 = \frac{1}{2}mH^2R^2$$

The total energy of the galaxy is the sum of the kinetic and potential energies

$$E = P.E. + K.E. = mR^2 \left[\frac{1}{2}H^2 - \frac{4}{3}\pi\rho G \right]$$

This quantity must remain constant as the universe expands.

If E is negative the galaxy can never escape to infinity, because at very great distances the potential energy becomes negligible, in which case the total energy is just the kinetic energy, which is always positive. On the other hand, if E is positive the galaxy can reach infinity with some kinetic energy left. Thus, the condition for the galaxy to have just barely escape velocity is that E vanish, which gives

$$\frac{1}{2}H^2 = \frac{4}{3}\pi\rho G$$

In other words, the density must take the value

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$$\rho_{\text{exp}}(t) = \frac{1}{H(t)} = \sqrt{\frac{3}{8\pi\rho(t)G}}$$

For instance, at the time of the first frame in Chapter V the mass density was 3.8 thousand million grams per cubic centimeter. Thus, the expansion time then was

$$t_{\text{exp}} = \sqrt{\frac{3}{8\pi(3.8 \times 10^9 \text{ gm/cm}^3)(6.67 \times 10^{-8} \text{ cm}^3/\text{gm sec}^2)}} = 0.022 \text{ seconds}$$

Now, how does $\rho(t)$ vary with $R(t)$? If the mass density is dominated by the masses of nuclear particles (the matter-dominated era), then the total mass within a comoving sphere of radius $R(t)$ is just proportional to the number of nuclear particles within that sphere, and hence must remain constant:

$$\frac{4\pi}{3}\rho(t)R(t)^3 = \text{constant}$$

Hence $\rho(t)$ is inversely proportional to $R(t)^3$

$$\rho(t) \propto 1/R(t)^3$$

(The symbol \propto means "is proportional to . . .") On the other hand, if the mass density is dominated by the mass equivalent to the energy of radiation (the radiation-dominated era), then $\rho(t)$ is proportional to the fourth power of the temperature. But the temperature varies like $1/R(t)$, so $\rho(t)$ is then inversely proportional to $R(t)^4$

$$\rho(t) \propto 1/R(t)^4$$

In order to be able simultaneously to deal with the matter- and radiation-dominated eras, we will write these results in the form

$$\rho(t) \propto [1/R(t)]^n$$

with

$$n = \begin{cases} 3 & \text{matter-dominated era} \\ 4 & \text{radiation-dominated era} \end{cases}$$

Note incidentally that $\rho(t)$ does blow up at least as fast as $1/R(t)^3$ for $R(t) \rightarrow 0$, as promised.

The Hubble constant is proportional to $\sqrt{\rho}$, and therefore

$$H(t) \propto [1/R(t)]^{n/2}$$

But the velocity of the typical galaxy is then

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$$V(t) = H(t)R(t) \propto [R(t)]^{1-n/2}$$

It is an elementary result of differential calculus that, whenever the velocity is proportional to some power of the distance, then the time that it takes to go from one point to another is proportional to the change in the ratio of distance to velocity. To be more specific, for V proportional to $R^{1-n/2}$, this relation is

$$t_1 - t_2 = \frac{2}{n} \left[\frac{R(t_1)}{V(t_1)} - \frac{R(t_2)}{V(t_2)} \right]$$

or

$$t_1 - t_2 = \frac{2}{n} \left[\frac{1}{H(t_1)} - \frac{1}{H(t_2)} \right]$$

We can express $H(t)$ in terms of $\rho(t)$, and find that

$$t_1 - t_2 = \frac{2}{n} \sqrt{\frac{3}{8\pi G}} \left[\frac{1}{\sqrt{\rho(t_1)}} - \frac{1}{\sqrt{\rho(t_2)}} \right]$$

Thus, whatever the value of n , the time elapsed is proportional to the change in the inverse square root of the density.

For instance, during the whole of the radiation-dominated era after the annihilation of electrons and positrons, the energy density was given by

$$\rho = 1.22 \times 10^{-38} [T(^{\circ} \text{K})]^4 \text{ gm/cm}^3$$

(See mathematical note 6, p. 176.) Also, here we have $n = 4$. Thus, the time required for the universe to cool from 100 million degrees to 10 million degrees was

$$\begin{aligned} t &= \frac{1}{2} \sqrt{\frac{3}{8\pi(6.67 \times 10^{-8} \text{ cm}^3/\text{gm sec})}} \\ &\times \left[\frac{1}{\sqrt{1.22 \times 10^{-38} \times 10^{28} \text{ gm/cm}^3}} - \frac{1}{\sqrt{1.22 \times 10^{-38} \times 10^{32} \text{ gm/cm}^3}} \right] \\ &= 1.90 \times 10^6 \text{ sec} = 0.06 \text{ years} \end{aligned}$$

Our general result can also be expressed more simply by saying that the time required for the density to drop to a value ρ from some value very much greater than ρ is

$$t = \frac{2}{n} \sqrt{\frac{3}{8\pi G\rho}} = \begin{cases} \frac{1}{2} t_{\text{exp}} & \text{radiation-dominated} \\ \frac{3}{2} t_{\text{exp}} & \text{matter-dominated} \end{cases}$$

(If $\rho(t_2) \gg \rho(t_1)$, we can neglect the second term in our formula for $t_1 - t_2$.) For instance, at 3,000° K the mass density of photons and neutrinos was

$$\rho = 1.22 \times 10^{-38} \times [3,000]^4 \text{ gm/cm}^3 = 9.9 \times 10^{-22} \text{ gm/cm}^3$$

This is so much less than the density at 10° K (or 10° K, or 10° K) that the time required for the universe to cool from very high early temperatures to 3,000° K may be calculated (setting $n = 4$) simply as

$$\begin{aligned} \frac{1}{2} \sqrt{\frac{3}{8\pi(6.67 \times 10^{-8} \text{ cm}^3/\text{gm sec}^2)(9.9 \times 10^{-22} \text{ gm/cm}^3)}} \\ = 2.1 \times 10^{13} \text{ sec} = 680,000 \text{ years} \end{aligned}$$

We have shown that the time required for the density of the universe to drop to a value ρ from much higher earlier values is proportional to $1/\sqrt{\rho}$, while the density ρ is proportional to $1/R^n$. The time is therefore proportional to $R^{n/2}$, or, in other words

$$R \propto t^{2/n} = \begin{cases} t^{1/2} & \text{radiation-dominated era} \\ t^{3/4} & \text{matter-dominated era} \end{cases}$$

This remains valid until the kinetic and potential energies have both decreased so much that they are beginning to be comparable to their sum, the total energy.

As remarked in Chapter II, there is at any time t after the beginning a horizon at a distance of order ct , from beyond which no information could yet have reached us. We now see that $R(t)$ vanishes less rapidly as $t \rightarrow 0$ than the distance to the horizon, so that at a sufficiently early time any given "typical" particle is beyond the horizon.

Note 4 Black-body Radiation

The *Planck distribution* gives the energy du of black-body radiation per unit volume, in a narrow range of wavelengths from λ to $\lambda + d\lambda$, as

$$du = \frac{8\pi hc}{\lambda^5} d\lambda / \left[e^{\left(\frac{hc}{kT\lambda} \right)} - 1 \right]$$

Here T is the temperature; k is Boltzmann's constant (1.38×10^{-16} erg° K); c is the speed of light (299,729 km/sec); e is the numerical constant 2.718

...; and h is the Planck constant (6.625×10^{-27} erg sec), originally introduced by Max Planck as an ingredient in this formula.

For long wavelengths, the denominator in the Planck distribution may be approximated by

$$e^{\left(\frac{hc}{kT\lambda} \right)} - 1 \approx \left(\frac{hc}{kT\lambda} \right)$$

Thus in this wavelength region, the Planck distribution gives

$$du = \frac{8\pi kT}{\lambda^4} d\lambda$$

This is the *Rayleigh-Jeans formula*. If this formula held down to arbitrarily small wavelengths, $du/d\lambda$ would become infinite for $\lambda \rightarrow 0$, and the total energy density in black-body radiation would be infinite.

Fortunately, the Planck formula for du reaches a maximum at a wavelength

$$\lambda = .2014052 \text{ hc/kT}$$

and then falls steeply off for decreasing wavelengths. The total energy density in the black-body radiation is the integral

$$u = \int_0^{\infty} \frac{8\pi hc}{\lambda^5} d\lambda / \left[e^{\left(\frac{hc}{kT\lambda} \right)} - 1 \right]$$

Integrals of this sort can be looked up in standard tables of definite integrals; the result is

$$u = \frac{8\pi^5 (kT)^4}{15 (hc)^3} = 7.56464 \times 10^{-15} [T(^{\circ} \text{K})]^4 \text{ erg/cm}^3$$

This is the *Stefan-Boltzmann law*.

We can easily interpret the Planck distribution in terms of quanta of light, or photons. Each photon has an energy given by the formula

$$E = hc/\lambda$$

Hence the number dN of photons per unit volume in black-body radiation in a narrow range of wavelengths from λ to $\lambda + d\lambda$ is

$$dN = \frac{du}{hc/\lambda} = \frac{8\pi}{\lambda^4} d\lambda / \left[e^{\left(\frac{hc}{kT\lambda} \right)} - 1 \right]$$

The total number of photons per unit volume is then

$$N = \int_0^{\infty} dN = 60.42198 \left(\frac{kT}{hc} \right)^3 = 20.28 [T(^{\circ} \text{K})]^3 \text{ photons/cm}^3$$

and the average photon energy is

$$E_{\text{average}} = u/N = 3.73 \times 10^{-16} [T(^{\circ} \text{K})] \text{ ergs}$$

Now let's consider what happens to black-body radiation in an expanding universe. Suppose the size of the universe changes by a factor f ; for instance, if it doubles in size, then $f = 2$. As we saw in Chapter II, the wavelengths will change in proportion to the size of the universe to a new value

$$\lambda' = f\lambda$$

After the expansion, the energy density du' in the new wavelength range λ' to $\lambda' + d\lambda'$ is less than the original energy density du in the old wavelength range λ to $\lambda + d\lambda$, for two different reasons:

1. Since the volume of the universe has increased by a factor f^3 , as long as no photons have been created or destroyed, the number of photons per unit volume has decreased by a factor $1/f^3$.

2. The energy of each photon is inversely proportional to its wavelength, and is therefore decreased by a factor $1/f$. It follows that the energy density is decreased by an overall factor $1/f^3$ times $1/f$, or $1/f^4$:

$$du' = \frac{1}{f^4} du = \frac{8\pi hc}{\lambda'^5} d\lambda / \left[e^{\left(\frac{hc}{kT\lambda'} \right)} - 1 \right]$$

If we rewrite this formula in terms of the new wavelengths λ' , it becomes

$$du' = \frac{8\pi hc}{\lambda'^5} d\lambda' / \left[e^{\left(\frac{hc}{kT\lambda'} \right)} - 1 \right]$$

But this is exactly the same as the old formula for du in terms of λ and $d\lambda$, except that T has been replaced with a new temperature

$$T' = T/f$$

Thus, we conclude that freely expanding black-body radiation remains described by the Planck formula, but with a temperature that drops in inverse proportion to the scale of the expansion.

Note 5 The Jeans Mass

In order for a clump of matter to form a gravitationally bound system, it is necessary for its gravitational potential energy to exceed its internal thermal

energy. The gravitational potential energy of a clump of radius r and mass M is of order

$$P.E. \approx -\frac{GM^2}{r}$$

The internal energy per unit volume is proportional to the pressure p , so the total internal energy is of order

$$I.E. \approx pr^3$$

Thus gravitational clumping should be favored if

$$\frac{GM^2}{r} \gg pr^3$$

But for a given density ρ we can express r in terms of M through the relation

$$M = \frac{4\pi}{3} \rho r^3$$

The condition for gravitational clumping may therefore be written

$$GM^2 \gg p(M/\rho)^{4/3}$$

or in other words

$$M \gg M_J$$

where M_J is (within an inessential numerical factor) the quantity known as the *Jeans mass*:

$$M_J = \frac{p^{3/2}}{G^{3/2} \rho^2}$$

For instance, just before the recombination of hydrogen, the mass density was $9.9 \times 10^{-22} \text{ gm/cm}^3$ (see mathematical note 3, p. 171), and the pressure was

$$p \approx \frac{1}{3} c^2 \rho = 0.3 \text{ gm/cm sec}^2$$

The Jeans mass was therefore

$$M_J = \left(\frac{0.3 \text{ gm/cm sec}^2}{6.67 \times 10^{-8} \text{ cm}^3/\text{gm sec}^2} \right)^{3/2} \left(\frac{1}{9.9 \times 10^{-22} \text{ gm/cm}^3} \right)^2 \\ = 9.7 \times 10^{81} \text{ gm} = 5 \times 10^{18} M_\odot$$

where M_\odot is one solar mass. (In comparison, the mass of our galaxy is about $10^{11} M_\odot$.) After recombination, the pressure dropped by a factor 10⁹, so the Jeans mass dropped to

$$M_J = (10^{-9})^{3/2} \times 5 \times 10^{18} M_\odot = 1.6 \times 10^8 M_\odot$$

It is interesting that this is roughly the mass of the large globular clusters within our galaxy.

Note 6 Neutrino Temperature and Density

As long as thermal equilibrium is preserved, the total value of the quantity known as "entropy" remains fixed. For our purposes, the entropy per unit volume S is given to an adequate approximation at temperature T by

$$S \propto N_T T^3$$

where N_T is the effective number of species of particles in thermal equilibrium whose threshold temperature lies below T . In order to keep the total entropy constant, S must be proportional to the inverse cube of the size of the universe. That is, if R is the separation between any pair of typical particles, then

$$SR^3 \propto N_T T^3 R^3 = \text{constant}$$

Just before the annihilation of electrons and positrons (at about 5×10^9 K) the neutrinos and antineutrinos had already gone out of thermal equilibrium with the rest of the universe, so the only abundant particles in equilibrium were the electron, positron, and photon. Referring to Table One on page 156, we see the effective total number of particle species before annihilation was

$$N_{\text{before}} = \frac{7}{2} + 2 = \frac{11}{2}$$

On the other hand, after the annihilation of electrons and positrons in the fourth frame, the only remaining abundant particles in equilibrium were the photons. The effective number of particle species then was simply

$$N_{\text{after}} = 2$$

It follows then from the conservation of entropy that

$$\frac{11}{2} (TR)^3_{\text{before}} = 2 (TR)^3_{\text{after}}$$

That is, the heat produced by the annihilation of electrons and positrons increases the quantity TR by a factor

$$\frac{(TR)_{\text{after}}}{(TR)_{\text{before}}} = \left(\frac{11}{4} \right)^{1/3} = 1.401$$

Before the annihilation of electrons and positrons, the neutrino temperature T_ν was the same as the photon temperature T . But from then on, T_ν simply dropped like $1/R$, so for all subsequent times $T_\nu R$ was equal to the value of TR before the annihilation:

$$(T_\nu R)_{\text{after}} = (T_\nu R)_{\text{before}} = (TR)_{\text{before}}$$

We conclude therefore that after the annihilation process is over, the photon temperature is higher than the neutrino temperature by a factor

$$\frac{(T/T_\nu)_{\text{after}}}{(T_\nu R)_{\text{after}}} = \frac{(TR)_{\text{after}}}{(T_\nu R)_{\text{after}}} = \left(\frac{11}{4} \right)^{1/3} = 1.401$$

Even though out of thermal equilibrium, the neutrinos and antineutrinos make an important contribution to the cosmic energy density. The effective number of species of neutrinos and antineutrinos is 7/2, or 7/4 of the effective number of species of photons. (There are two photon spin states.) On the other hand, the fourth power of the neutrinos' temperature is less than the fourth power of the photon temperature by a factor $(4/11)^{4/3}$. Thus the ratio of the energy density of neutrinos and antineutrinos to that of photons is

$$\frac{u_\nu}{u_\gamma} = \frac{7}{4} \left(\frac{4}{11} \right)^{4/3} = 0.4542$$

The Stefan-Boltzmann law (see Chapter III) tells us that at photon temperature T the photon energy density is

$$u_\gamma = 7.5641 \times 10^{-15} \text{ erg/cm}^3 \times [T(\text{K})]^4$$

Hence the total energy density after electron-positron annihilation is

$$u = u_\nu + u_\gamma = 1.4542 u_\gamma = 1.100 \times 10^{-14} \text{ erg/cm}^3 [T(\text{K})]^4$$

We can convert this to an equivalent mass density by dividing by the square of the speed of light, and find

$$\rho = u/c^2 = 1.22 \times 10^{-35} \text{ gm/cm}^3 \times [T(\text{K})]^4$$